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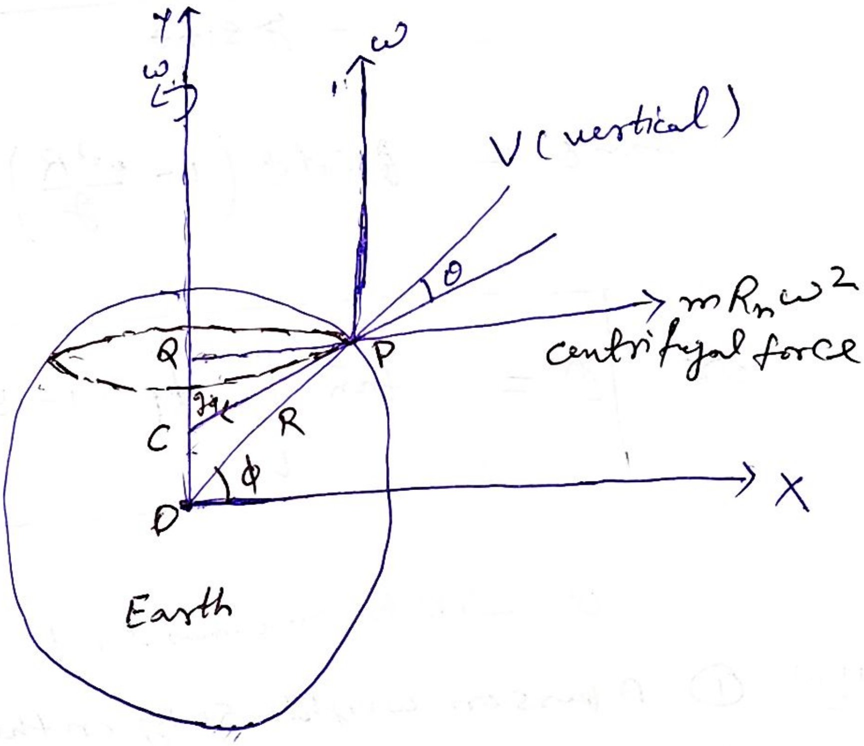
Effect of Coriolis force due to Earth's rotation:

In the last lecture note, we have discussed effect of centrifugal force due to Earth's rotation. Here we discuss the effect of Coriolis force.



If we consider that ~~also~~ a body is in motion with respect to the rotating frame of reference of the earth, then the Coriolis force comes into picture and we have two interesting cases:

(i) When body is dropped from rest ~~and~~ so that it falls freely under the action of the gravitational force

(ii) When the body is given a large horizontal velocity → (This is like a case of projectile motion)

Case-(i):- → Horizontal component have little effect in this case on the freely falling body i.e., it ~~defel~~ deflects a little from its vertical position.

→ Vertical component only affects the value of g.

Next, to calculate the displacement or deflection of a freely falling body, we consider that coordinates axes are denoted by x, y and z and are taken along the east, north and vertically upward directions respectively. The corresponding unit vectors are denoted by $\hat{i}$, $\hat{j}$ and $\hat{k}$ respectively.

velocity $\vec{v}$ ~~after~~ gained by body in time 't' is given by

$$\vec{v} = -v\hat{k}$$ (-ve sign shows that direction of velocity is downward)

$$\vec{\omega} = \omega\cos\phi\,\hat{j} + \omega\sin\phi\,\hat{k}$$ { ϕ → latitude

Coriolis acceleration ($\vec{a}_c$) given by

$$\vec{a}_c = -2\vec{\omega}\times\vec{v}.$$

$$= -2\omega(\cos\phi\,\hat{j} + \sin\phi\,\hat{k})\times(-v\hat{k})$$

$$= 2\omega v\cos\phi\,\hat{i} - 0$$

$$\vec{a}_c = 2\omega v\cos\phi\,\hat{i}$$

Or $$\frac{d^2x}{dt^2} = 2\omega v\cos\phi$$

v → velocity gained by particle in time t,

$$v = 0 + gt = gt$$

$$\therefore \frac{d^2x}{dt^2} = 2\omega gt\cos\phi$$

Now the x-component of the velocity (V_x) is given by $V_x = \frac{dx}{dt} = \int 2\omega g t \cos\phi \, dt$

or $V_x = 2\omega g \cos\phi \frac{t^2}{2} + C_0$

cont. of integ.

$V_x = \omega g \cos\phi \, t^2 + C_0$

at $t=0$, $V_x = 0 \rightarrow C_0 = 0$

$$\boxed{V_x = \omega g \cos\phi \, t^2}$$

for displacement $\frac{dx}{dt} = V_x = \omega g \cos\phi \, t^2$

$x = \int \omega g \cos\phi \, t^2 dt$

or $x = \omega g \cos\phi \frac{t^3}{3} + C_1$

at $t=0 \rightarrow x=0 \Rightarrow C_1 = 0$

$$\therefore \boxed{x = \frac{1}{3}\omega g \cos\phi \, t^3}$$

Here, t is the ~~time~~ time taken by the body to fall through a height h. Therefore, $h = \frac{1}{2}gt^2 \Rightarrow t = \sqrt{2h/g}$

~~Now~~ $\cancel{x = \frac{1}{3}\omega g \cos\phi (2gh)^{3/2}}$

Now $x = \frac{1}{3}\omega g \cos\phi \left(\frac{2h}{g}\right)^{3/2}$

or $$\boxed{x = \left(\frac{8}{9g}\right)^{\frac{1}{2}} h^{\frac{3}{2}} \omega \cos\phi}$$

Thus, the horizontal displacement of the body due to the Coriolis force in ~~[illegible]~~ latitude ϕ is $\left(\frac{8}{9g}\right)^{\frac{1}{2}} h^{\frac{3}{2}} \omega \cos\phi$

at $\phi = 0$, i.e., at equator, $(\cos\phi = 1)$ $\quad x = \left(\frac{8}{9g}\right)^{\frac{1}{2}} h^{3/2} \omega$

$\Rightarrow$ Displacement is maximum at equator. It is directed along +ve x-axis, i.e. east direction.

Case-(ii) :- In this case horizontal velocity is large $\rightarrow$ the body covers large horizontal distance. The position vector due to Coriolis force is given by $-\omega \sin\phi$.

In Northern hemisphere $\rightarrow \phi = +$ve $\Rightarrow$ Rotation viewed from above is clockwise and projectile gets deflected towards right

In Southern hemisphere $\rightarrow \phi = -$ve $\Rightarrow$ Rotation anticlockwise and projectile deflection towards left.

H.W. ① A mass of 1 Kg is hurled horizontally due north with a ~~[illegible]~~ velocity of 0.5 Km per second in latitude $30^\circ N$. Obtain the magnitude and direction of the Coriolis force acting on the ~~[illegible]~~ mass and its deflection in consequence thereof in a time interval of half a second.